Revised zone method *R*-value calculation for precast concrete sandwich panels containing metal wythe connectors

Byoung-Jun Lee
and Stephen Pessiki

Editor's quick points

- Metal wythe connectors are used in a typical precast concrete sandwich panel to tie concrete wythes together and to keep the panel intact during handling and in service.
- This paper proposes a new zone-width equation for use in the current zone method to compute the *R*-value of precast concrete sandwich panels containing metal wythe connectors.
- The proposed zone-width equation can effectively consider the effects of metal-wythe-connector sizes and spacing, material conductivities, and panel thicknesses in the zone method of *R*-value computation.

In a typical precast concrete sandwich panel, wythe connectors are used to tie the two concrete wythes together and to keep the panel intact during handling and in service. The wythe connectors pass from one concrete wythe to the other through the insulation layer. Thus, the connectors interrupt the continuous insulation layer, causing thermal bridges. Depending on the material used to make the connectors, these thermal bridges can conduct energy at a much higher rate than the insulation, thus reducing the effectiveness of the insulation.[1] According to McCall,[2] the thermal performance of a panel may be reduced by as much as 40% by the large amount of heat conducted through the concrete regions and the wythe connectors that penetrate the insulation.

Figure 1 shows a typical precast concrete sandwich panel. A sandwich panel is often described by a three-digit sequence of numbers, in which each digit denotes the thickness of one of the layers, or wythes, in the panel. For example, a 3-2-3 panel comprises two 3-in.-thick (76 mm) concrete wythes separated by a 2-in.-thick (51 mm) insulation layer. The panel shown in Fig. 1 contains solid concrete regions and metal wythe connectors. **Figure 2** shows typical metal wythe connectors. Their spacing typically varies from 16 in. × 16 in. (406 mm × 406 mm) to 48 in. × 48 in. (1212 mm × 1212 mm).[3]

The thermal resistance, or *R*-value, of a material or assembly of materials is a quantity that is often used to describe the thermal performance of building construction.[3,4] An *R*-value calculation for a sandwich panel includes analyzing the panel for the effects of thermal bridges.

In current practice, the computation of an *R*-value for a precast concrete sandwich panel is based on the zone method given in the American Society of Heating, Refrigerating and Air-Conditioning Engineers' *ASHRAE Handbook: Fundamentals* (ASHRAE handbook)[5] and is summarized in the *PCI Design Handbook: Precast, Prestressed Concrete*.[6] As explained in the "Background" section of this paper, a key parameter in the zone method is the zone width *W*, which is calculated from the geometry of the construction. However, the zone-width parameter *W* was originally developed for metal-frame structures. Application of the method to treat metal wythe connectors in precast concrete sandwich panels leads to erroneous results.

Lee and Pessiki[7,8] proposed a method called the *characteristic section method* to compute *R*-values for the precast concrete sandwich panels, in which solid concrete regions function as thermal bridges. This method is now included in the *PCI Design Handbook*. However, the characteristic section method only includes thermal bridges caused by solid concrete regions, not those created by metal wythe connectors.

This paper proposes a new zone-width equation, Eq. (3), for use in the current zone method to compute an *R*-value of a precast concrete sandwich panel that contains metal wythe connectors. The proposed zone width W_n was derived by considering the results of a series of finite element heat-transfer analyses intended to quantify the influence of several key parameters on W_n. A panel was modeled using the finite element method (FEM). A new zone width was back-calculated from the zone method in such a way that the FEM *R*-value was the same as that obtained from the zone method. Based on a series of analyses, it was found that the proposed zone width W_n is a function of wythe connector size and material conductivities.

The proposed zone width can be used with the zone method and the characteristic section method to compute the thermal *R*-values for precast concrete sandwich wall panels that contain both metal wythe connectors and solid concrete regions.

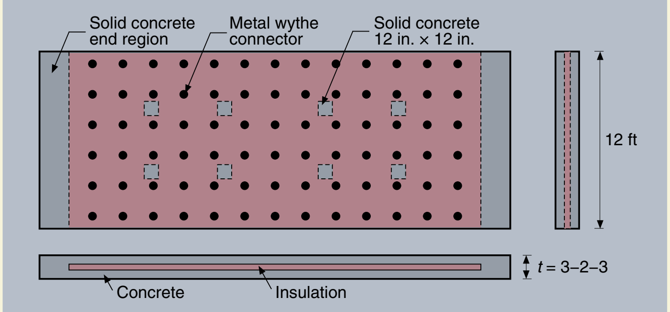

Figure 1. The diagram depicts a typical precast concrete sandwich panel. Note: 3-2-3 is in inches. 1 in. = 25.4 mm; 1 ft = 0.3048 m.

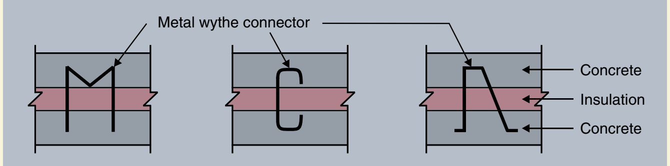

Figure 2. This panel cross-section view shows three typical metal wythe connectors.

Background

ASHRAE handbook methods, experimental methods, and the FEMs can be used to estimate *R*-values of precast concrete sandwich panels. ASHRAE handbook and experimental methods are briefly summarized here, and the next section describes the FEM that was used to understand and quantify the impacts of key parameters in deriving the proposed zone width.

ASHRAE *R*-value estimates

The ASHRAE handbook describes three methods to compute *R*-values through a material or assembly of materials using electric-circuit analogies. These methods are the parallel flow, isothermal plane, and zone methods. In these methods, the thermal resistances of the materials are treated as electrical resistances that are arranged in parallel, a series, or a combination of the two analogies to estimate an *R*-value of the assembly. The ASHRAE handbook gives descriptions of each method, but only the zone method is described in this paper.

Zone method The zone method can be used to compute the *R*-value of an assembly when it contains widely spaced, high-thermal-conductivity elements of a substantial cross-sectional area. The zone method involves two separate computations: one for a chosen, limited-portion zone A, containing the highly conductive element, and the other for the remaining-portion zone B, of simpler construction. The two zones are combined after separate computations are made. A key parameter in the zone method is the zone width *W*.

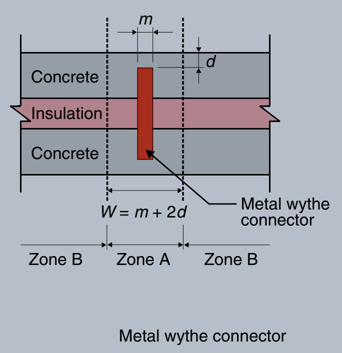

Figure 3. The zone method uses this simplified panel geometry.

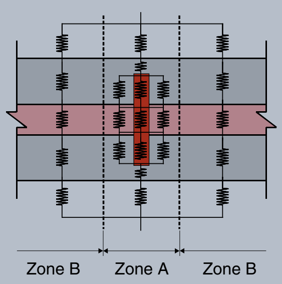

Figure 4. This figure shows an electrical circuit analogy for the zone method.

Figure 3 illustrates the application of the zone method to a portion of a precast concrete sandwich panel containing one metal wythe connector. When the metal wythe connector contains two legs, the legs can be combined into a single element. Figure 3 shows the panel geometry with the legs of a metal connector combined as a single element. The portion shown can be considered to represent a larger panel with metal wythe connectors placed at regular spacing. For the given panel, Eq. (1) calculates zone A's width *W*.

$$W = m + 2d \tag{1}$$

where

m = diameter of the connector

d = distance from the panel surface to the connector

In Eq. (1), d should not be less than 0.5 in. (13 mm) for still air.[5]

Figure 4 illustrates the electric circuit analogy for the geometry illustrated in Fig. 3. Thermal resistances are computed in series for each separate zone. The two zones are then combined in parallel. In Fig. 4, the thermal resistance outside of the panel represents an air-film resistance. The appendix illustrates a sample zone-method calculation.

Experimental method

The guarded hot box method is a general test method that can be used to estimate the thermal performance of assemblies such as sandwich panels.[9] In the guarded hot box method, the test panel is placed inside the guarded hot box and exposed to hot air on one side and cold air on the other side. Testing is performed by establishing and maintaining a desired steady air-temperature difference across a test panel for a period of time. As will be described in a following section, the guarded hot box method was modeled using FEM to estimate the *R*-value of the panel.

FEM *R*-value estimate

FEM heat-transfer analyses were performed to understand and quantify the impact of key parameters in deriving the proposed zone width W_n. The next section describes a complete analysis procedure to obtain the proposed zone-width equation and a procedure to obtain FEM *R*-values for precast concrete sandwich panels.

A panel being studied was modeled using finite elements for conditions present in the guarded hot box test. The FEM *R*-value was calculated from the results of the analysis. Lee and Pessiki give complete details of the FEM analysis.[10,11] All FEM heat-transfer analyses were executed using the SAP 90 heat-transfer analysis program.

FEM model

Figure 5 shows the FEM model used to estimate the *R*-value of a panel. A typical panel exhibits a repeated panel geometry with respect to wythe connector spacing, so only a small portion of the panel containing one metal wythe connector was studied. In the case shown in Fig. 5, the metal wythe connectors were spaced at 24 in. (610 mm) on center. Then, considering symmetric boundary conditions, a one-quarter-symmetry model was treated in the FEM model as shown in Fig. 5.

Three-dimensional heat-transfer analyses were performed to estimate the *R*-values of the panels. The concrete, insulation, and metal wythe connectors were modeled with eight-node solid-brick elements. The temperature variation over these elements was linear.

As shown in Fig. 5, only convection and conduction were considered in the FEM model. Convection heat transfer occurs from the hot air to the surface of the panel according to the relationship

$Q = -h_h(t_2 - t_h)$

where

Q = heat flow

h_h = convection coefficient

t_2 = surface temperature on the hot side of the panel

t_h = ambient air temperature on the hot side of the panel

Inside the test panel, heat is transferred in conduction. The governing equation for conduction heat transfer is

$Q = -k(A\Delta T)$

where

k = material conductivity

A = material area

ΔT = temperature difference

Finally, convection heat transfer occurs again from the panel to the cold air according to the relationship

$Q = -h_c(t_1 - t_c)$

where

h_c = convection coefficient

t_1 = surface temperature on the cold side of the panel

t_c = ambient air temperature on the cold side of the panel

Radiation effects are not included because these are minimized by the materials used to construct the guarded hot box facilities.

Mesh refinement studies were performed to determine the appropriate FEM element size and aspect ratio to use in the analyses. The impact on *R*-values as a function of element size and aspect ratio were examined to arrive at final element sizes and shapes.

Figure 5 shows a typical FEM mesh used in this study. The cross section of the metal wythe connector was modeled as square rather than a circular cross section that is typical in practice. Thus, the metal wythe connector diameter referred in the FEM model was an equivalent diameter that represented the same cross-sectional area with the square cross section in the FEM model.

Material conductivities All materials were treated as isotropic with constant conductivity. The concrete conductivity k_{con} was taken as 12.05 (BTU × in.)/(hr × ft^2 × °F)[1.74 W/(m × °C)], which corresponds to a concrete density of 150 lb/ft^3 (23.6 kN/m^3) according to McCall.[2] Expanded polystyrene material with a conductivity k_{in} of 0.26 (BTU × in.)/(hr × ft^2 × °F)[0.037 W/(m × °C)] was assumed for the insulation. The metal wythe connector was assumed to be made of steel with a conductivity k_{ct} of 314.4 (BTU × in.)/(hr × ft^2 × °F)[45.3 W/(m × °C)]. These three conductivity values were used for all FEM analyses unless

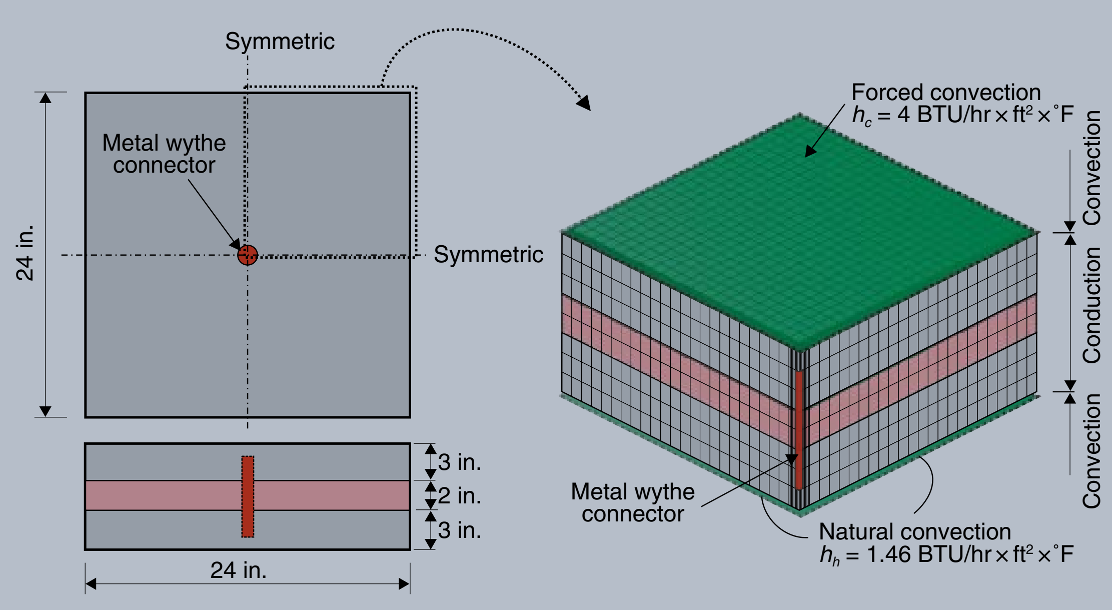

Figure 5. This finite element method model and mesh are used to estimate the *R*-value of a panel. Note: BTU = British thermal unit. 1 in. = 25.4 mm; 1 ft = 0.3048 m; 1 BTU/(hr × ft^2 × °F) = 5.68 W/(m^2 × °C).

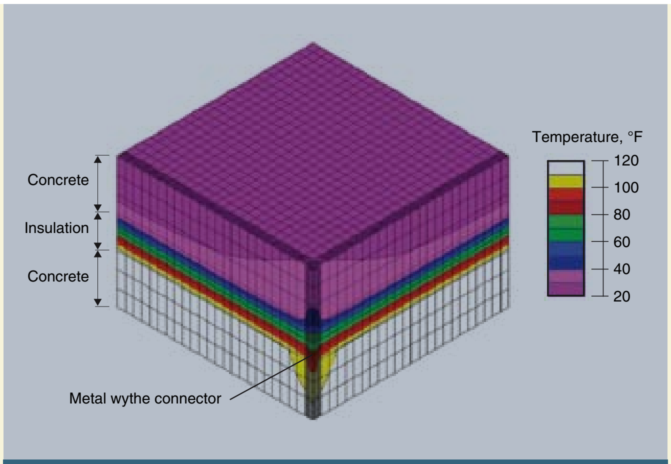

Figure 6. Shown is the temperature distribution for the precast concrete sandwich panel containing a metal wythe connector. Note: °F = (°C × 1.8) + 32.

they were intentionally varied as a parameter to study the influence of material conductivity.

Boundary conditions Convection boundaries were specified, as shown in Fig. 5, on both surfaces of the panel. The convection boundaries also functioned as loading, and were modeled as shell elements. A forced convection boundary was specified for the cold surface of the panel with a convection coefficient h_c of 4 BTU /(hr × ft^2 × °F) [22.7 W/(m^2 × °C)]. A natural convection boundary was specified for the hot surface of the panel with a convection coefficient h_h of 1.46 BTU/(hr × ft^2 × °F)[8.3 W/(m^2 × °C)]. All convection coefficients were determined according to the ASHRAE handbook. Finally, an adiabatic surface was assumed where a symmetry boundary condition existed.

R-value calculation

The temperature and heat-flow results of the analysis were used to compute an *R*-value. Equation (2) determines the thermal resistance *R*.

Table 1. Variable limitations in Eq. (3)

Variable	Range
Metal wythe connector diameter *m*, in.	0 to 0.85
Concrete conductivity k_{con}, $\frac{\text{BTU}\times\text{in.}}{\text{hr}\times\text{ft}^2\times{}^\circ\text{F}}$	3.6 to 20.5
Insulation conductivity k_{in}, $\frac{\text{BTU}\times\text{in.}}{\text{hr}\times\text{ft}^2\times{}^\circ\text{F}}$	0.1 to 0.36
Wythe connector conductivity k_{ct}, $\frac{\text{BTU}\times\text{in.}}{\text{hr}\times\text{ft}^2\times{}^\circ\text{F}}$	94 to 346

Note: BTU = British thermal unit. 1 in. = 25.4 mm;
$1\ \frac{\text{BTU}\times\text{in.}}{\text{hr}\times\text{ft}^2\times{}^\circ\text{F}} = 0.1442\ \frac{\text{W}}{\text{m}\times{}^\circ\text{C}}$.

$$R = \frac{A\left(t_h - t_c\right)}{Q} \tag{2}$$

In building applications, the thermal resistance given by Eq. (2) is often called the air-to-air *R*-value. When using panel surface temperatures instead of air temperatures in Eq. (2), the thermal resistance of the panel is called surface-to-surface *R*-value. All *R*-values presented in this paper are air-to-air *R*-values.

Verification of the FEM model

The FEM approach used in this research was developed and verified in earlier work. In that work, the results of guarded hot box tests to measure *R*-values of three different wall systems were compared with FEM analyses of the same three wall systems. The three wall systems included a sandwich wall panel containing regions of solid concrete, a sandwich wall panel without any concrete thermal bridges, and a concrete block wall with core insulation. *R*-values estimated using the FEM analyses agreed with the *R*-values obtained in the experiments. Lee and Pessiki present complete details.[10,11]

Temperature distribution

The FEM heat-transfer analysis provided the temperature at each node for all of the elements. This temperature distribution helped to understand the manner in which heat transferred through the panel.

Figure 6 shows a typical temperature distribution in a precast concrete sandwich panel that includes a metal wythe connector. The panel geometry shown was the same as that shown in Fig. 5. The quarter-symmetry model included a 0.35-in.-diameter (8.9 mm) metal wythe connector with a cover distance to the metal wythe connector of 1.0 in. (25 mm). The ambient air temperatures on opposite faces of the panel were 25 °F (-3.9 °C) and 125 °F (51.7 °C). The given metal wythe connector diameter represented the same area as one conventional M-tie that has two $^1/_4$-in.-diameter (6.4 mm) legs.

As shown in Fig. 6, the panel surface temperatures deviated from the average surface temperatures at the metal wythe connector location. This metal wythe connector clearly functioned as a thermal bridge. Also, lateral heat transfer occurred in the panel near the metal wythe connector location, as can be seen from the temperature contours (the direction of heat flow is perpendicular to the temperature contours).

Proposed zone width W_n

The influence of metal wythe connectors on the *R*-value of a precast concrete sandwich panel can be computed using the zone method. However, it is proposed that the zone width W_n for a precast concrete sandwich panel be calculated with Eq. (3) instead of Eq. (1).

$$W_n = (0.174k_{con} - k_{in} + 0.0026k_{ct} + 2.24)m + 0.02k_{con} - 0.6k_{in} + 0.0024k_{ct} + 2.35 - 0.15d \quad (3)$$

where

k_{con} = concrete conductivity

k_{in} = insulation conductivity

k_{ct} = metal wythe connector conductivity

Equation (3) is applicable within the range of variables in **Table 1**.

Various panel geometries, connector geometries, and material conductivities were investigated when deriving Eq. (3) by the zone method *R*-value calculation. For a typical precast concrete sandwich panel with the conductivities given in the previous section, Eq. (3) becomes Eq. (4).

$$W_n = 4.9m + 3.5 - 0.15d \quad (4)$$

Derivation of proposed zone width

Equation (3) was derived from parametric studies. A panel being studied was modeled using FEM, and the temperatures and heat flow from the analysis were used to compute the *R*-value for the panel. The zone width was back calculated from the zone method in such a way that the FEM *R*-value was the same as that obtained from the zone method. The parametric studies included changes in wythe connector sizes and spacing and material conductivities. The zone width was back calculated for each analysis case. After numerous zone-width computations, it was found that the zone width is a function of wythe connector size and material conductivities. Also, the material conductivities are coupled with the wythe connector size. Equation (5) was assumed from these conclusions.

$$W_n = (C_1k_{con} + C_2k_{in} + C_3k_{ct} + C_4)m + C_5k_{con} + C_6k_{in} + C_7k_{ct} + C_8 + C_9d \quad (5)$$

where

C_n = undetermined constants

In Eq. (5), constants C_1 through C_9 were determined from the parametric studies of different wythe connector sizes and cover distance, with varying material conductivities.

Application of proposed zone width

As described previously, various metal wythe connectors are used in practice. There are different applications of Eq. (3) for these various connectors.

A metal wythe connector that has two legs can be treated to have one leg with an equivalent diameter that represents the total area of the two-leg metal wythe connector. This will overestimate *R*-value, but the error is small because typical metal wythe connectors have a small diameter leg. If the two legs are far enough apart that the W_n for each leg does not overlap, they may be treated individually instead of combining the two legs into one leg.

The horizontal portion of the metal wythe connector has little effect on *R*-value and can be ignored in the *R*-value calculation.

A metal wythe connector that has a slanted leg can be treated as one that has a vertical leg.

The appendix shows an example calculation of the zone method with the proposed zone width W_n.

Parametric studies

Parametric studies were performed for various metal wythe connector sizes, spacings, material conductivities, and panel thicknesses. *R*-values were computed from the zone method with the proposed zone width given in Eq. (3), hereafter referred to as R_N-values. They were compared with the zone method *R*-values computed using the original zone width given in Eq. (1), hereafter referred to as R_O-values, and FEM *R*-values, hereafter referred to as R_{FEM}-values.

The parametric studies focused on a 3-2-3 prototype panel with 0.346-in.-diameter (8.8 mm) metal wythe connectors spaced at 24 in. (610 mm) on center. The cover distance *d* was 1.0 in. (25 mm). Material properties were the same as described previously. The parameters used in the prototype panel were default values in the parametric studies, and a selected parameter was varied for each parametric study.

Metal wythe connector configuration

Figure 7 shows a plot of normalized *R*-values versus metal wythe connector spacing *s*. The metal wythe connector spacing varied from 12 in. to 36 in. (305 mm to 914 mm), and R_N-, R_O-, and R_{FEM}-values were calculated. All *R*-values were then normalized by the *R*-value of a panel that does not contain any thermal bridge. The *R*-value of such a panel is 9.16 (hr × ft^2 × °F)/(BTU) [1.61(m^2 × °C)/(W)]. The R_N-values computed from the zone method with the proposed zone width agreed well with the R_{FEM}-values. In contrast, the R_O-values were consistently higher than the R_{FEM}-values.

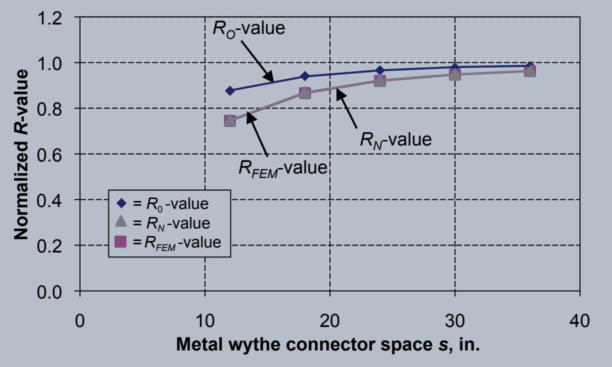

Figure 7. This graph compares the normalized *R*-value with the metal wythe connector spacing. Note: 1 in. = 25.4 mm.

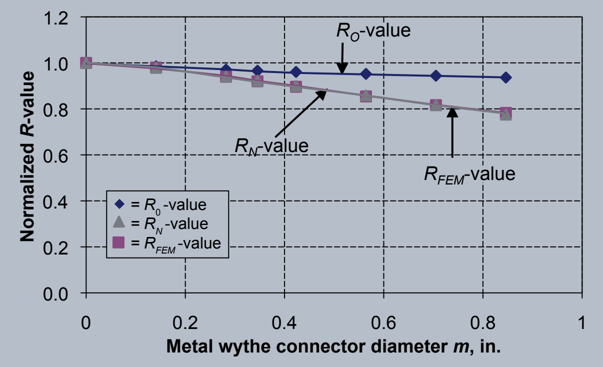

Figure 8. This graph compares the normalized *R*-value with the metal wythe connector diameter. Note: 1 in. = 25.4 mm.

Figure 8 shows a plot of normalized *R*-values versus metal wythe connector diameter *m*. The metal wythe connector diameter varied from 0 in. to 0.85 in. (0 mm to 21.6 mm). Similar to Fig. 7, *R*-values were estimated three different ways and then normalized by the *R*-value of a panel without any thermal bridge. Again, the R_N-values agreed well with the R_{FEM}-values, while the R_O-values were consistently higher.

Figure 9 shows a plot of normalized *R*-values versus cover distance *d*. The cover distance varied from 0 in. to 3.0 in. (0 mm to 76 mm). The R_N-values agreed well with R_{FEM}-values, while the R_O-values do not effectively consider the effect of the cover distance.

Material conductivity

The conductivities of the concrete, insulation, and metal wythe connector were varied to examine how the zone method with the proposed zone width predicts *R*-values. In this material-conductivity variation, one selected material conductivity was systematically varied while the other two conductivities were kept constant. A large range of material conductivities was selected to include materials that are typically used in practice.

Figure 10 shows the relationship between *R*-value and concrete conductivity k_{con}. Unlike the previous plots, these *R*-values were not normalized. As expected, the *R*-values decreased when the concrete conductivity increased. The R_N-values were almost the same as the R_{FEM}-values, while the R_O-values were 3% to 6% higher than the R_{FEM}-values.

Figure 11 shows the relationship between *R*-value and insulation conductivity k_{in}. As expected, the *R*-values decreased when the insulation conductivity increased. The R_N-values agreed well with the R_{FEM}-values. Alternatively, the R_O-values were 3% to 13% higher than the R_{FEM}-values.

Figure 12 shows the relationship between *R*-value and metal wythe connector conductivity k_{ct}. Similar to previous comparisons, the R_N-values agreed well with the R_{FEM}-values, but the R_O-values were again higher.

Panel thickness

Tables 2 and **3** compare *R*-values when varying panel wythe thickness. Table 2 shows panels with symmetric wythe thicknesses, and Table 3 shows panels with nonsymmetric wythe thicknesses. Both tables include R_{FEM}-,

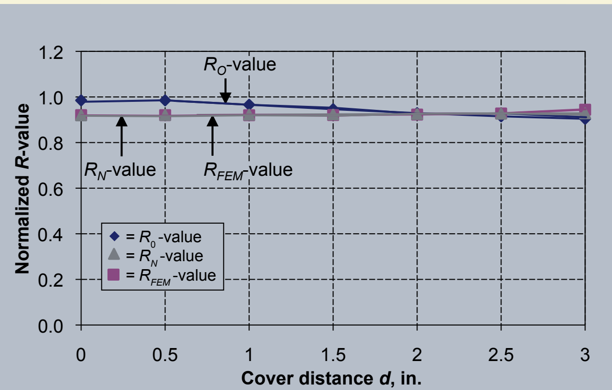

Figure 9. This graph compares the normalized *R*-value with the cover distance. Note: 1 in. = 25.4 mm.

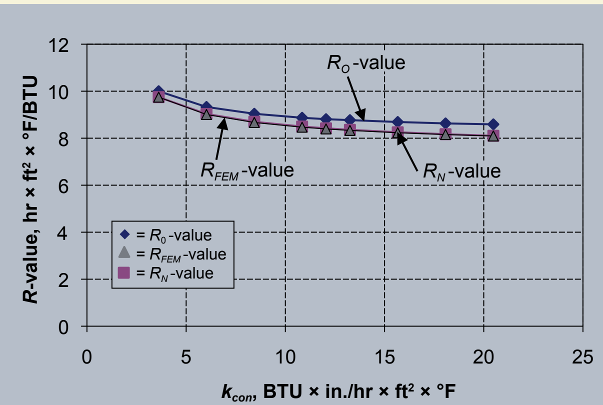

Figure 10. This graph compares the *R*-value with the concrete conductivity. Note: BTU = British thermal unit. 1 (hr × ft² × °F)/ BTU = 0.1761 (m² × °C)/W; 1 (BTU × in)/ (hr × ft² × °F) = 0.1442 W/(m × °C).

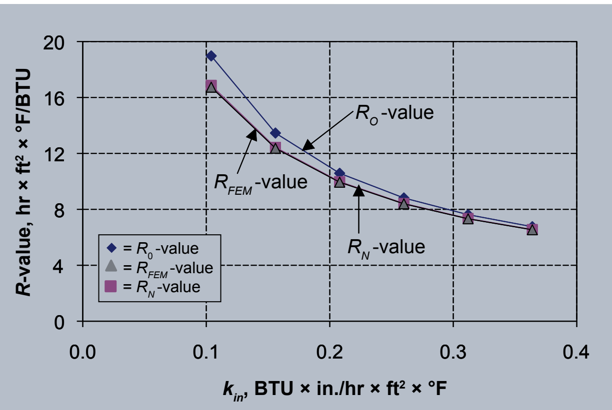

Figure 11. This graph compares the *R*-value with the insulation conductivity. Note: BTU = British thermal unit. 1 (hr × ft² × °F)/ BTU = 0.1761 (m² × °C)/W; 1 (BTU × in.)/ (hr × ft² × °F) = 0.1442 W/(m × °C).

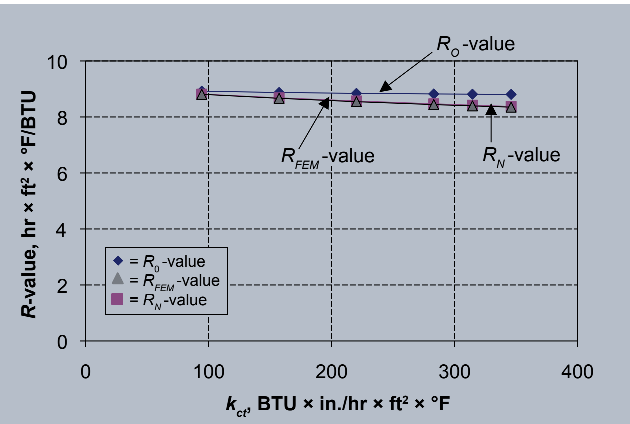

Figure 12. This graph compares the *R*-value with the metal wythe conductivity. Note: BTU = British thermal unit. 1 (hr × ft² × °F)/ BTU = 0.1761 (m² × °C)/W; 1 (BTU × in.)/ (hr × ft² × °F) = 0.1442 W/(m × °C).

R_O-, and R_N-values for each case. The R_{FEM}-values were used to normalize the R_O-values and R_N-values.

The R_N-values agreed well with the R_{FEM}-values—more so than the R_O-values did.

Conclusion

Three major conclusions were developed based on the analyses presented:

- The ASHRAE handbook zone method with the proposed zone-width equation can accurately estimate the *R*-values of precast concrete sandwich panels containing metal wythe connectors. The proposed zone-width equation is applicable within the range of variables shown in Table 1.
- The proposed zone-width equation can effectively consider the effects of metal wythe connector sizes and spacing, material conductivities, and panel thicknesses in the zone method of *R*-value computation.
- The current zone method with the original zone-width equation predicts higher *R*-values than the FEM *R*-values.

Acknowledgments

This research was funded by the Center for Advanced Technology for Large Structural Systems (ATLSS) at Lehigh University. The Precast/Prestressed Concrete Institute is a general partner of the ATLSS Center. The support from ATLSS and PCI is gratefully acknowledged. The findings and conclusions presented in this paper are those of the authors, and do not necessarily reflect the views of the sponsors.

Table 2. *R*-values for symmetric thickness panels

Panel wythe thickness, in.-in.-in.	Finite element method	Zone method with Eq. (1)		Zone method with Eq. (3)	
	R_{FEM}-value, $\frac{hr \times ft^2 \times °F}{BTU}$	R_O-value, $\frac{hr \times ft^2 \times °F}{BTU}$	R_O/R_{FEM}	R_N-value, $\frac{hr \times ft^2 \times °F}{BTU}$	R_N/R_{FEM}
2-1-2	4.9	5.0	1.03	4.8	0.99
3-1-3	5.0	5.2	1.03	5.0	1.00
4-1-4	5.2	5.3	1.03	5.2	1.00
2-2-2	8.3	8.6	1.04	8.2	0.99
3-2-3	8.4	8.8	1.05	8.4	1.00
4-2-4	8.5	9.0	1.05	8.6	1.01
2-3-2	11.6	12.2	1.05	11.5	1.00
3-3-3	11.7	12.4	1.06	11.7	1.00
4-3-4	11.8	12.6	1.06	12.0	1.01

Note: 1 in. = 25.4 mm; $1\ \frac{hr \times ft^2 \times °F}{BTU} = 0.1761\ \frac{m^2 \times °C}{W}$.

Table 3. *R*-values for non-symmetric thickness panels

Panel wythe thickness, in.-in.-in.	Finite element method	Zone method with Eq. (1)		Zone method with Eq. (3)	
	R_{FEM}-value, $\frac{hr \times ft^2 \times °F}{BTU}$	R_0-value, $\frac{hr \times ft^2 \times °F}{BTU}$	R_0/R_{FEM}	R_N-value, $\frac{hr \times ft^2 \times °F}{BTU}$	R_N/R_{FEM}
2-1-2	4.9	5.0	1.03	4.8	0.99
3-1-3	4.9	5.1	1.03	4.9	1.00
4-1-4	5.0	5.2	1.03	5.0	1.00
2-2-2	8.3	8.6	1.04	8.2	0.99
3-2-3	8.3	8.7	1.05	8.3	1.00
4-2-4	8.4	8.8	1.06	8.4	1.01
2-3-2	11.6	12.2	1.05	11.5	1.00
3-3-3	11.6	12.3	1.06	11.6	1.00
4-3-4	11.7	12.4	1.06	11.7	1.00

Note: BTU = British thermal unit. 1 in. = 25.4 mm; $1\ \frac{hr \times ft^2 \times °F}{BTU} = 0.1761\ \frac{m^2 \times °C}{W}$.

References

1. Gabor, L. 1971. Thermal Steady State Analysis. *Build International*, V. 4, No. 1 (January–February): pp. 58–63.

2. McCall, W. C. 1985. Thermal Properties of Sandwich Panels. *Concrete International*, V. 7, No. 1: pp. 35–41.

3. PCI Committee on Precast Sandwich Wall Panels. 1997. State-of-the-Art of Precast/Prestressed Sandwich Wall Panels. *PCI Journal,* V. 42, No. 2 (March–April): pp. 92–134.

4. Holman, J. P. 2002. *Heat Transfer*. 9th ed. New York, NY: McGraw-Hill.

5. American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE). 1997. *ASHRAE Handbook: Fundamentals*. I-P ed. Atlanta, GA: ASHRAE.

6. PCI Industry Handbook Committee. 2004. *PCI Design Handbook: Precast and Prestressed Concrete*. 6th ed. Chicago, IL: PCI.

7. Lee, Y. J. 2003. Development of the Characteristic Section Method to Estimate *R*-values of Precast Concrete Sandwich Wall Panels. MS thesis, Department of Civil and Environmental Engineering, Lehigh University, Bethlehem, PA.

8. Lee, Y. J., and S. Pessiki. 2003. *Development of the Characteristic Section Method to Estimate* R-*values of Precast Concrete Sandwich Wall Panels*. ATLSS report no. 03-06. Bethlehem, PA: Lehigh University.

9. ASTM Committee C-16. 1989. ASTM C236-89: Standard Test Method for Steady-State Thermal Performance of Building Assemblies by Means of a Guarded Hot Box. In *Annual Book of ASTM Standards*, V. 04.06. West Conshohocken, PA: ASTM.

10. Lee, B. J. 2003. Development of a Precast Prestressed Concrete Three-Wythe Sandwich Wall Panel. PhD diss. Department of Civil and Environmental Engineering, Lehigh University, Bethlehem, PA.

11. Lee, B. J., and S. Pessiki. 2004. Analytical Investigation of Thermal Performance of Precast Concrete Three-wythe Sandwich Wall Panels. *PCI Journal*, V. 49, No. 4 (July–August): pp. 88–101.

Notation

A = panel area

A_f = panel area fraction

C_n = undetermined constant

d = distance from panel surface to metal

E_z = affected zone in characteristic section method

h_c = convection film coefficient for cold side of panel

h_h = convection film coefficient for hot side of panel

k = material conductivity

k_{con} = concrete conductivity

k_{ct} = metal wythe connector conductivity

k_{in} = insulation conductivity

m = width or diameter of metal wythe connector

Q = heat flow

R = thermal resistance

t = panel thickness

t_1 = surface temperature of cold side

t_2 = surface temperature of hot side

t_c = ambient air temperature for cold side of panel

t_h = ambient air temperature for hot side of panel

U = thermal transmittance

W = zone A width in zone method

W_n = proposed zone A width in zone method

α = insulation conductivity coefficient factor in characteristic section method

β = conductivity coefficient factor in characteristic section method

ΔT = temperature gradient

Appendix: Calculation of *R*-values for precast concrete sandwich wall panels with thermal bridges

Example 1: Compute the *R*-value of the panel containing M-ties

k_{con} = concrete conductivity

$$= 13.33 \frac{\text{BTU} \times \text{in.}}{\text{hr} \times \text{ft}^2 \times {}^\circ\text{F}} \left(1.74 \frac{\text{W}}{\text{m} \times {}^\circ\text{C}}\right)$$

k_{in} = insulation conductivity

$$= 0.20 \frac{\text{BTU} \times \text{in.}}{\text{hr} \times \text{ft}^2 \times {}^\circ\text{F}} \left(0.029 \frac{\text{W}}{\text{m} \times {}^\circ\text{C}}\right)$$

k_{ct} = metal wythe conductivity

$$= 314.4 \frac{\text{BTU} \times \text{in.}}{\text{hr} \times \text{ft}^2 \times {}^\circ\text{F}} \left(45.3 \frac{\text{W}}{\text{m} \times {}^\circ\text{C}}\right)$$

Panel wythe thickness = 3-2-3, in inches (75-50-75, in mm)

Solution

M-ties are regularly spaced, so consider one $^1/_4$-in.-diameter M-tie in a 24 in. × 24 in. (610 mm × 610 mm) portion of the sandwich panel (**Fig. A.1** and **A.2**). Calculate zone width W_n in zone A.

Combining two legs of M-tie to one leg, the equivalent bar diameter *m* is

$$m = \sqrt{2}(1/4) = 0.354 \text{ in. (9 mm)}$$

d = M-tie cover distance = 1.0 in. (25 mm)

According to Eq. (3),

$$W_n = (0.174k_{con} - k_{in} + 0.0026k_{ct} + 2.24)m + 0.02k_{con} - 0.6k_{in} + 0.0024k_{ct} + 2.35 - 0.15d$$

$$W_n = [(0.174 \times 13.33) - 0.20 + (0.0026 \times 314.4) + 2.24](0.354) + (0.02 \times 13.33) - (0.6 \times 0.20) + (0.0024 \times 314.4) + 2.35 - (0.15 \times 1.0) = 4.93 \text{ in. (125 mm)}$$

$$\text{Zone A area} = \frac{\pi W_n^2}{4}$$

$$= (3.14)(4.93)^2/4 = 19.1 \text{ in.}^2 \text{ (12,300 mm}^2\text{)}$$

$$\text{Zone B area} = (24)(24) - 19.1 = 556.9 \text{ in.}^2 \text{ (359,300 mm}^2\text{)}$$

Computing the *R*-value of the panel treating zone A and zone B in parallel:

Fractional area of zone A: 19.1/(24 × 24) = 0.033

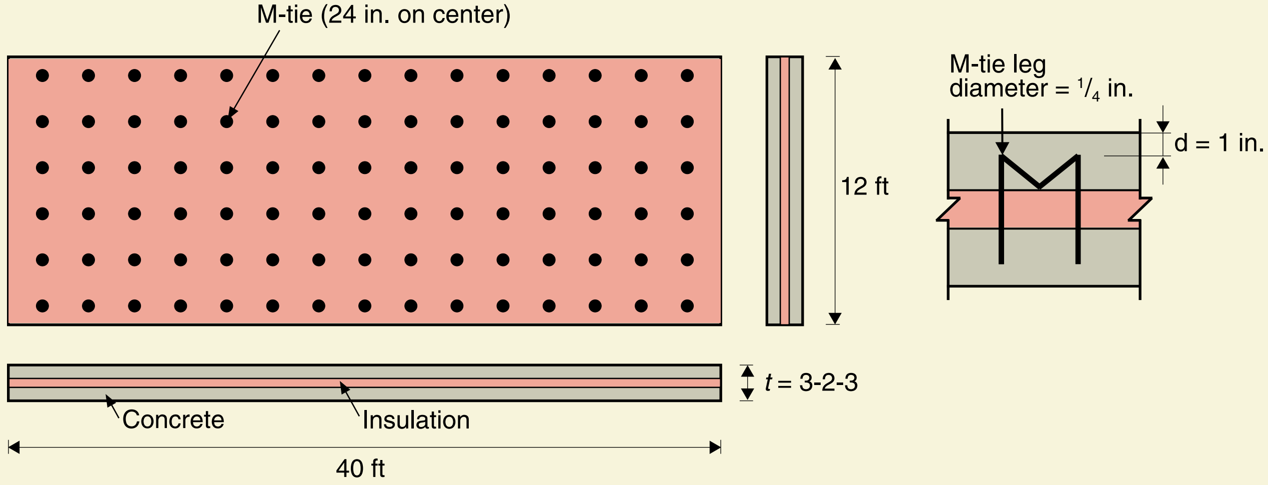

Figure A.1. This figure shows a 3-2-3 precast concrete sandwich panel. The concrete wythes are connected by M-ties. Note: Drawing is not to scale. 3-2-3 is in inches. 1 in. = 25.4 mm; 1 ft = 0.3048 m.

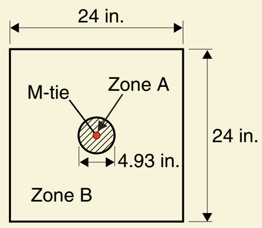

Figure A.2. This diagram shows a 24 in. × 24 in. portion of a precast concrete sandwich panel containing one M-tie. Note: 1 in. = 25.4 mm.

Fractional area of zone B: 556.9/(24 × 24) = 0.967

Winter:

$$\frac{1}{R} = \frac{0.033}{2.38} + \frac{0.967}{11.31} = 0.099$$

$$R = 10.06 \text{ hr} \times \text{ft}^2 \times °\text{F/BTU } (1.77 \text{ m}^2\cdot°\text{C/W})$$

Similarly, R = 10.17 hr × ft^2 × °F/BTU (1.79 m^2·°C/W) in summer.

Table A.1. *R*-value in zone A

Component	Area fraction A_f	Conductivity k, $\frac{\text{BTU} \times \text{in.}}{\text{hr} \times \text{ft}^2 \times °\text{F}}$	Thickness t, in.	$U = A_j k / t$	$R = 1/\Sigma U$ winter	$R = 1/\Sigma U$ summer
Outside surface	n.a.	n.a.	n.a.	n.a.	0.17	0.25
Concrete	1.0	13.33	1	13.33	0.075	0.075
Concrete M-tie	0.995 0.005	13.33 314.4	2 2	6.63 0.79	0.13	0.13
Insulation M-tie	0.995 0.005	0.20 314.4	2 2	0.10 0.79	1.12	1.12
Concrete M-tie	0.995 0.005	13.33 314.4	2 2	6.63 0.79	0.13	0.13
Concrete	1.0	13.33	1	13.33	0.075	0.075
Inside surface	n.a.	n.a.	n.a.	n.a.	0.68	0.68
Total					**2.38**	**2.46**

Note: n.a. = not applicable. 1 in. = 25.4 mm; 1 $\frac{\text{BTU} \times \text{in.}}{\text{hr} \times \text{ft}^2 \times °\text{F}} = 0.1442 \frac{\text{W}}{\text{m} \times °\text{C}}$.

Table A.2. *R*-value in zone B

Component	Conductivity k, $\frac{\text{BTU} \times \text{in.}}{\text{hr} \times \text{ft}^2 \times °\text{F}}$	Thickness t, in.	$U = k / t$	$R = 1/\Sigma U$ winter	$R = 1/\Sigma U$ summer
Outside surface	n.a.	n.a.	n.a.	0.17	0.25
Concrete	13.33	3	4.44	0.23	0.23
Insulation	0.20	2	0.10	10.00	10.00
Concrete	13.33	3	4.44	0.23	0.23
Inside surface	n.a.	n.a.	n.a.	0.68	0.68
Total				**11.31**	**11.39**

Note: n.a. = not applicable. 1 in. = 25.4 mm; 1 $\frac{\text{BTU} \times \text{in.}}{\text{hr} \times \text{ft}^2 \times °\text{F}} = 0.1442 \frac{\text{W}}{\text{m} \times °\text{C}}$.

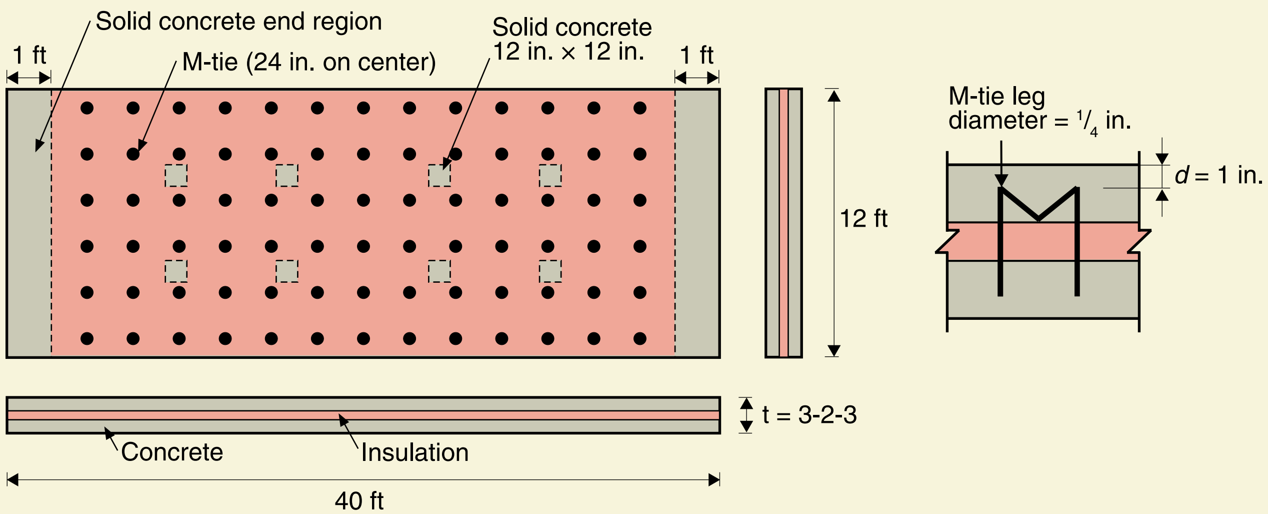

Figure A.3. This figure shows a 3-2-3 precast concrete sandwich panel. Its concrete wythes are connected by M-ties and areas of solid concrete. Note: Drawing is not to scale. 3-2-3 is in inches. 1 in. = 25.4 mm; 1 ft = 0.3048 m.

Example 2: Compute the *R*-value of the panel containing M-ties and solid concrete regions

Solution

Consider three thermal paths through the panel shown in **Fig. A.3**: (1) zone A containing M-ties; (2) through solid concrete regions; and (3) perfect insulated path.

R-value through solid concrete regions (refer to *PCI Design Handbook*[1] example 9.1.8.1):

Using characteristic section method

Parameters: $\alpha = 0.48$, $\beta = 1.15$, $E_z = 2.7$ in. with given material conductivities

Fractional area of the solid concrete region to the panel:

$$\frac{(2)(12+2.7)(144)+(8)(12+2\times2.7)(12+2\times2.7)}{(40\times12)(144)} = 0.096$$

Computing the *R*-value of the panel treating the paths (1), (2), and (3) in parallel:

Winter:

$$\frac{1}{R} = \frac{0.033}{2.38}+\frac{0.096}{1.45}+\frac{0.871}{11.31} = 0.157$$

$$R = 6.37\ \frac{\text{hr}\times\text{ft}^2\times{}^\circ\text{F}}{\text{BTU}}\ (1.12\ \frac{\text{m}^2\times{}^\circ\text{C}}{\text{W}})$$

Similarly, $R = 6.55\ \frac{\text{hr}\times\text{ft}^2\times{}^\circ\text{F}}{\text{BTU}}\ (1.15\ \frac{\text{m}^2\times{}^\circ\text{C}}{\text{W}})$ in summer.

Table A.3. *R*-value through solid concrete regions

Component	Conductivity k, $\frac{\text{BTU}\times\text{in.}}{\text{hr}\times\text{ft}^2\times{}^\circ\text{F}}$	Thickness t, in.	$U = k/t$	$R = 1/\Sigma U$ winter	$R = 1/\Sigma U$ summer
Outside surface	n.a.	n.a.	n.a.	0.17	0.25
Concrete	13.33	8	1.67	0.60	0.60
Inside surface	n.a.	n.a.	n.a.	0.68	0.68
Total				**1.45**	**1.53**

Note: n.a. = not applicable. 1 in. = 25.4 mm; $1\ \frac{\text{BTU}\times\text{in.}}{\text{hr}\times\text{ft}^2\times{}^\circ\text{F}} = 0.1442\ \frac{\text{W}}{\text{m}\times{}^\circ\text{C}}$.

Table A.4. Summarizing *R*-values for each thermal path

Path	Area fraction A_f	R_{FEM}-value, $\frac{hr \times ft^2 \times °F}{BTU}$	
		Winter	Summer
Zone A containing M-ties	0.033	2.38*	2.46*
Solid concrete regions	0.096	1.45	1.53
Perfect insulated path	0.871	11.31*	11.39*

* Computed in example 1

Note: BTU = British thermal unit. 1 $\frac{hr \times ft^2 \times °F}{BTU} = 0.1761 \frac{m^2 \times °C}{W}$.

References

1, PCI Industry Handbook Committee. 2004. *PCI Design Handbook: Precast and Prestressed Concrete*. 6th ed. Chicago, IL: PCI.

About the authors

Byoung-Jun Lee, PhD, is a staff engineer for Whitlock Dalrymple Poston & Associates PC in Manassas, Va.

Stephen Pessiki, PhD, is a professor and chair of the Department of Civil and Environmental Engineering at Lehigh University in Bethlehem, Pa.

Synopsis

Metal wythe connectors are used in a typical precast concrete sandwich panel to tie concrete wythes together and to keep the panel intact during handling and in service. Connectors interrupt the continuous insulation layer, reducing the effectiveness of the insulation. In current practice, thermal resistance (*R*-value) of such a panel is calculated from the zone method. However, the zone width parameter W used in the zone method was originally developed for metal-frame structures and an accurate *R*-value cannot be estimated for precast concrete sandwich panels containing metal wythe connectors.

This paper proposes a new zone-width equation for use in the current zone method to compute the *R*-value of precast concrete sandwich panels containing the metal wythe connectors. The proposed zone width W_n was derived from the results of a series of finite element heat-transfer analyses intended to quantify the influence of several key parameters on W_n. It was found that the zone method with the proposed zone-width equation can accurately estimate *R*-values of a precast concrete sandwich panel containing metal wythe connectors. Also, the proposed zone-width equation can effectively consider the effects of metal wythe connector sizes and spacing, material conductivities, and panel thicknesses in the zone method of *R*-value computation.

Keywords

Precast concrete, *R*-value, sandwich panel, thermal analysis, wythe connector, zone method.

Review policy

This paper was reviewed in accordance with the Precast/Prestressed Concrete Institute's peer-review process.

Reader comments

Please address any reader comments to *PCI Journal* editor-in-chief Emily Lorenz at elorenz@pci.org or Precast/Prestressed Concrete Institute, c/o *PCI Journal*, 209 W. Jackson Blvd., Suite 500, Chicago, IL 60606.