

# RELIABILITY OF PRESTRESSED CONCRETE BRIDGE STRUCTURES

Reynaldo M. Pablo Jr., PhD

Department of Manufacturing & Construction Engineering Technology and Interior Design  
Indiana University-Purdue University Fort Wayne  
Fort Wayne, IN

## ABSTRACT

*Highway bridges usually possess reserved strength to accommodate occasional overloads although they were designed for standard loads. It is commonly practiced that the strength of a bridge component is allowed to be higher than what is required by the design specifications. Depending on a number of factors, this additional amount of strength may be substantial. However, many countries allow overloads on their highway systems. In Michigan, for example, vehicles exceeding the national truck weight limit are allowed to cross the bridges. This paper addresses this issue and examines the safety and reliability of selected prestressed concrete bridges in Michigan. Safety and reliability of these bridges are assessed using the reliability index  $\beta$ . Results show that the current design load does not consistently achieve the target reliability level.*

**Keywords:** : Bridge Safety, Structural Reliability, Reliability Index

## INTRODUCTION

Structural safety has traditionally been described and quantified in terms of factors of safety. The theory of structural reliability instead quantifies structural safety using a measurement of risk, taking into account the uncertainty involved. It is also worth mentioning that structural safety is time variant. It is because the load demand and the capacity of a structure may change over time. For example, many developed countries are currently experiencing a problem of aging and deteriorated bridge networks. These structures' safety has been of concern also due to observed growth of load in both magnitude and volume. Evaluation, repair, and rehabilitation are necessary for the preservation of the load capacity and service performance of these existing bridges. To minimize cost of replacement or repair, the evaluation needs to accurately reveal the current load carrying capacity of the bridge and to cover future loads and further changes in the capacity. Note that this involves a significant amount of uncertainty. To this end, the reliability theory of structures can be a helpful tool to quantify the risk involved in this process of bridge assessment.

This study examined fifteen prestressed concrete bridges from the bridge inventory of the Michigan Department of Transportation (MDOT). The bridges were selected randomly from a suite of bridges constructed or re-constructed after 1990 in the state of Michigan. This bridge suite included the following three superstructure types: 1) prestressed concrete I-beam bridges (PI), 2) prestressed concrete spread box beam bridges (PCS), and 3) prestressed concrete adjacent box beam bridges (PCA). Accordingly, these three superstructure types represents current and foreseeable future population of new prestressed concrete bridges in the state. Five bridges of each type were evaluated for this study.

Safety and reliability of the bridges were assessed using the reliability-based algorithms that measure the safety reserve in a structure covering the focused uncertainty involved. The concept of structural reliability was used for the assessment of the bridges. Bridge reliability was measured using the structural reliability index  $\beta$ , which has been used in several recent research projects related to bridge safety<sup>1,2,3</sup>, including NCHRP Project 12-33 Development of LRFD Bridge Design Specifications. In that project, the LRFD bridge design code was calibrated with respect to structural reliability index  $\beta$ . The design load was examined in the context of the load and resistance factor design (LRFD) following requirements of the LRFD bridge design code<sup>4</sup>. The target reliability index of 3.5 for calibrating the AASHTO LRFD Bridge Design Specifications was used as the criterion for evaluating the reliability of the bridges.

## **STRUCTURAL RELIABILITY OF BRIDGE STRUCTURES**

The reliability of a structure is defined here as its probability to fulfill the safety requirement for a specified period. An important component of structural reliability is concerned with the calculation or estimation of the probability of a limit state violation for the structure during its lifetime. The probability of occurrence of structural failure or a limit state violation is a numerical measure of the likelihood of its occurrence. Its estimate may be obtained using

measurements of the long-term frequency of occurrence of the interested event for generally similar structures, or using numerical analysis and simulation. Reliability estimates for structures are often obtained using analysis and simulation, based on measurement data for the elements involved in modeling. For example, for highway bridge structures, statistics of data for these elements are used in modeling, such as bridge components' strengths, sizes, deterioration rates, truck load magnitudes, traffic volume, etc.

The likelihood that a random variable may take a particular value is described by its probability distribution function<sup>5</sup> or cumulative distribution function (CDF) and probability density function (PDF). The most important characteristic parameters of a random variable are its mean value or average value, standard deviation, and probability distribution type. The standard deviation gives a measure of dispersion or variability. The standard deviation of a random variable  $R$  with a mean  $\mu_R$  is often symbolized as  $\sigma_R$ . A dimensionless measure of the variability is the coefficient of variation (COV) which is the ratio between the standard deviation and the mean value,  $\sigma_R / \mu_R$ .

In this study, the margin of safety for a bridge component can be defined as

$$Z = R - S \quad (1)$$

where  $R$  is the resistance or the load carrying capacity of the structural component, and  $S$  is the load effect or the load demand to the component. They are modeled as random variables here because their uncertainty is evident. In general, the uncertainty associated with the resistance is due to material production and preparation process, construction quality control, etc. The uncertainty associated with load effect is related to truck weight, truck type, traffic volume, etc.

The probability of failure  $P_f$  is the probability that the resistance  $R$  is lower than the total applied load  $S$ :

$$P_f = P_r[R \leq S] = P_r[Z \leq 0] = \int_{-\infty}^0 f_z(z) dz = F_z(0) \quad (2)$$

where  $P_r[E]$  is the probability of occurrence of the event  $E$ ,  $f_z(z)$  is the probability density of the variable  $Z$ , and  $F_z(0)$  is the value of the CDF for  $Z$  at  $Z=0$ . Thus, the probability of failure is obtained by summing the probabilities that  $Z$  has an outcome smaller than 0. It is also represented by the cumulative probability distribution function  $F_z(0)$ . Note that the failure probability in Eq. (2) refers to a load effect in a structural component. Hence, this definition can be applied to a variety of load effects, such as moment, shear, and even possibly displacement. It also can be applied to a variety of bridge structural components, such as beams, slabs, and piers.

When the probability densities of  $R$  and  $S$  are available, Eq. (2) can also be expressed as:

$$P_f = \int_{-\infty}^{+\infty} F_R(x) f_s(x) dx \quad (3)$$

where  $f_s(x)$  and  $F_R(x)$  are the PDF of  $S$  and the CDF of  $R$ , respectively.

The structural reliability is defined as the probability that  $R$  is greater than  $S$  (or  $Z$  greater than  $0$ ). It is also called probability of survival  $P_s$  and thus defined as the complement of the probability of failure:

$$P_s = 1 - P_f \quad (4)$$

Structural safety can be measured by structural reliability index  $\beta^6$ . The reliability index  $\beta$  is defined as follows using Eq. (2)

$$\beta = \phi^{-1}(1 - P_f) \quad (5)$$

where  $\phi^{-1}(\cdot)$  is the inverse function of the standard normal random variable's CDF. Eq. (5) indicates that  $\beta$  is inversely monotonic with  $P_f$ . That is to say, a small  $P_f$  leads to a large  $\beta$ , or a large  $P_f$  to a small  $\beta$ . Thus a large  $\beta$  indicates a safer structural component and a small  $\beta$  a less safe one.

This study used this structural reliability concept to evaluate structural reliability of the selected prestressed concrete bridges in Michigan. The target reliability index of 3.5 was used in this study. This value was arbitrarily selected to provide the same average safety margin in the LRFD code. Note that the target level of 3.5 was selected not as an absolute criterion but rather a relative norm in the AASHTO LRFD code calibration process as the average of  $\beta$  levels.

## MODELING OF DEAD AND LIVE LOAD EFFECT STATISTICS

Modeling the effects of bridge loads is not a trivial task mainly because it requires measurement data to cover their variation over a long period of time. Very often such data are not available. Thus, it may require the prediction or projection of future loads, using measurement data collected over a shorter time period. Therefore, bridge load modeling is often associated with a certain degree of subjective judgment of uncertainty. It is important, however, to note that "the objective of load modeling is not to come up with an exact mathematical formulation of the loads and their effects, but to develop models to represent the most salient features of the loading phenomenon"<sup>7</sup>.

### DEAD LOAD EFFECT STATISTICS

Although the dead load of a bridge system is not considered to vary significantly with time, the actual value of the load is uncertain. In addition, the analysis of the dead load effect on the bridge structure also involves a degree of uncertainty, for example, due to the assumptions about the structural members' boundary conditions, etc. For many bridges, for example, the primary dead load is due to the weight of the primary beams, the deck, and the deck's surface. The uncertainties in predicting the magnitude of the dead load are due to variations in the density of the materials used to form the deck and other members as well as the variations in the dimensions of these members.

The dead load effect's nominal values were estimated using the available bridge plans of the fifteen sample bridges provided by the MDOT. The dead load was assumed to act as a uniformly distributed load to the focused bridge member. The critical beam in the bridge was selected to be the girder adjacent to the edge girder, i.e., the first interior girder. Those dead loads that were the result of safety railing or safety barriers located on the bridge edge were assumed to be distributed to the critical girder with a one-third factor. A 25 psf future wearing surface was also included in the dead load effect  $D$ . Each dead load has an associated bias and coefficient of variation (COV). The COV was defined as the ratio of the standard deviation to the mean value. The dead load bias  $D_{bias}$  was expressed in terms of the nominal dead load effect  $D_{nom}$  and the mean dead load effect  $D_{mean}$  as

$$D_{bias} = \frac{D_{mean}}{D_{nom}} \quad (6)$$

The bias and COV for the dead load effect was taken as 1.0 and 0.1, respectively<sup>1</sup>. Since the nominal value of a dead load effect was estimated according to the bridge's plans, the mean value of the dead load effect was readily obtained by multiplying the nominal value by the bias.

## LIVE LOAD EFFECT STATISTICS

There are a large number of transient loads that are normally applied on highway bridges. These include all moving loads as well as temperature effects, wind and earthquake loads. For typical short to medium span bridges, the most important loads are those due to moving vehicular traffic including their static and dynamic effects. Although these two effects occur simultaneously as one or more vehicles move across the bridge structure, it has been traditional in bridge engineering practice to treat the static effect separately from the dynamic effect. With this approach, bridge members are analyzed for the static effect of the vehicles and then a dynamic amplification factor is used to account the effect of bridge vibrations due to moving vehicles.

There are two existing methods for measuring weights of trucks. One is static weighing and the other is dynamic weighing. Static weighing involves stopping a vehicle and measuring

its weight statically, the weight of which is termed as static weight. Dynamic weighing allows a vehicle to be weighed while in motion, or dynamically, the weight of which is termed as weigh-in-motion (WIM) weight. Dynamic weighing through high-speed WIM systems provides continuous unbiased weighing of practically all vehicles passing the system. They are also hardly noticeable that the drivers are not aware of the weighing operation and do not try to avoid it. WIM scales are dynamic weighing systems that determine weights while vehicles are in motion. They enable vehicles to be weighed with little or no interruption of their travel. WIM scales have been designed to sense the weights of the axles passing over the instrument through the use of piezo sensors, strain gauges, or hydraulic or pneumatic pressure transducers. The readings are transmitted to a receiving unit, where they are converted to actual weights<sup>8</sup>. Dynamic truck weighing is more advantageous than static weighing. For this reason, only WIM data are to be reported to the Federal Highway Administration (FHWA) Truck Weight Study. Hence, WIM data become readily available from state Department of Transportation (DOT).

WIM data were used as live load of the bridge structures under investigation. Some WIM data for Michigan have been made available for this research for 5 functional classes (FC). These functional classifications included; FC01: Principal Arterial – Interstate – Rural, FC02: Principal Arterial – Other – Rural, FC11: Principal Arterial – Interstate – Urban, FC12: Principal Arterial – Urban, and FC14: Other Principal Arterial – Urban.

A total of over 46,000 trucks were included in this new data set, resulting in over 250,000 axle weights. This number of trucks was slightly reduced by eliminating a few that appear to be inaccurate. This included trucks recorded to have two axles weighing less than 10 kips and three or more axles weighing less than 15 kips. In a typical structural reliability analysis one is more concerned with the upper tail of the load. Thus, this “cleaning” does not noticeably affect the final result of reliability. It was found out from the live load effect data that a lognormal distribution was a good fit. Note that there is no 75 year WIM data collected yet that can be used to analyze reliability of bridges for its lifespan. Sometimes available WIM data were collected only over a period of several days for each site. In order to perform the reliability analysis for the entire lifespan of the bridges, it is necessary to project the live load effect (moment or shear), to the expected bridge life (75 years).

For modeling flexure and shear effect of truck live load (moving load), moment and shear influence lines were developed first for each bridge’s critical sections. Each influence line for a particular section and a particular load effect was used individually to obtain live load effect data for that section and load effect. Then every truck in the WIM dataset was “run” through the influence line to find the truck’s maximum load effect, using a computer program. The input parameters of the computer program are the influence lines and WIM dataset. For each influence line, after all the trucks in the WIM dataset had been used in this simulation process, a set of maximum live load effects was obtained to generate the statistics for that load effect. The results of maximum load effect for all the trucks consequently provided a set of data for modeling the random variable of that load effect.

Once the live load effect statistics for each critical section on each bridge was determined as described the data was projected to a 75-year statistical distribution. The following approach was used for this projection.

First, an equivalent number of days of data (*EDD*) was determined using the following equation:

$$EDD = \frac{m}{ADTT} \quad (7)$$

where  $m$  is the number of trucks in the dataset used for a case of reliability analysis and  $ADTT$  is the average daily truck traffic for the focused bridge site. Essentially  $EDD$  indicates the equivalent days of WIM data used for the particular site focused in the reliability analysis.

Secondly, an empirical CDF was constructed by sorting the dataset from smallest to largest load effects for the  $m$  trucks included in the dataset. The corresponding value of the CDF for the  $i^{\text{th}}$  ranked load effect can be expressed as

$$F_{i, \text{for } EDD \text{ days}} = \frac{\sum_{j \leq i} n_j}{m} = Prob[L < L_i]_{\text{for } EDD \text{ days}} \quad (8)$$

where  $n_j$  is the number of trucks including load effects falling in the  $j^{\text{th}}$  interval of the CDF. Thus,  $F_i$  is the cumulative probability for the load effect  $L$  to be lower than the  $i^{\text{th}}$  interval represented by  $L_i$ .

Thirdly, the projected CDF of  $L$  for 75 years was then obtained using the  $EDD$  defined in Eq. (7) and the number of  $EDD$  in 75 years,  $N$ , as

$$N = \frac{(75 \text{ years})(365 \text{ days / year})}{EDD} \quad (9)$$

The projected CDF,  $F_{i,75}$  was estimated using

$$F_{i,75} = F_i^N \quad (10)$$

This computation was based on an assumption that each time period of duration  $EDD$  within the time period of 75 years are statistically independent from one another.

## MODELING OF BRIDGE BEAM RESISTANCE STATISTICS

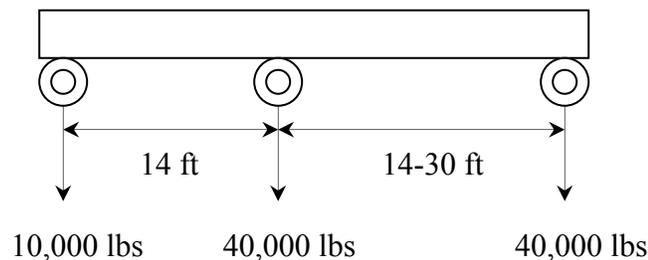
The resistance here refers to the moment or shear capacity of the bridge component being focused. Two different cases were considered; the as-built capacity and the as-designed capacity. The capacities were calculated at the critical sections of the bridge spans.

### AS-BUILT CAPACITY

To calculate the as-built capacity of the bridge, basic principles of engineering structural analysis/structural mechanics were used. In this case, the bridge plans used for construction were reviewed, and moment or shear capacities were computed. The determined values were taken as nominal resistance for probabilistic modeling. It should be noted that in the calculation of as-built capacities, no resistance factors (i.e., strength reduction factors) were applied.

### AS-DESIGNED CAPACITY

The as-designed capacity is based on AASHTO Standard Specifications<sup>9</sup> used in MDOT. The following requirements were followed. The HS25 design load consists of a truck load with the axle weights and spacing shown in Figure 1, or a lane load of 0.8 kip/ft plus one or more point loads of 22.5 kips (for moment) or 32.5 kips (for shear). Whichever gave a larger result was used as the nominal live load moment (or shear) for computing the required nominal moment (or shear) strength.



**Figure 1 Axle weights and spacing for HS25 bridge design load**

The design requirement according to AASHTO Standard Specifications was expressed in terms of moments (or shears) as a combined dead and live load effects as follows.

$$\phi R = 1.3(D) + 2.17(L + I) \quad (11)$$

where  $D$  and  $(L + I)$  are the nominal load effects due to dead load and live load plus dynamic impact factor, respectively,  $R$  is the nominal strength, and  $\phi$  is the resistance factor.

## RELIABILITY INDEX CALCULATIONS

For the reliability assessment of bridge components, the safety margin in Eq. (1) was further detailed as

$$Z = R - (D + L) \quad (12)$$

where  $(D + L) = S$ .  $D$  and  $L$  are respectively dead and live load effects. Live load here refers to truck load effect to the bridge component. Both  $D$  and  $L$  were also modeled as random variables. In order to estimate the reliability index for the bridges, it was necessary to estimate the statistical distributions for the load effects as well as the structural resistance. The mean and COV of the total load effect  $S$  were derived from the mean and COV of the dead load effect  $D$  and live load effect  $L$ .

Assuming that  $D$  and  $L$  were statistically independent of each other, the standard deviation  $\sigma_S$  was expressed as:

$$\sigma_S^2 = \sigma_D^2 + \sigma_L^2 \quad (13)$$

where  $\sigma$  is the standard deviation, and subscripts  $S$ ,  $D$ , and  $L$  are respectively for total, dead, and live load effect. The mean value for the total load effect  $S$  was then the sum of the means of  $D$  and  $L$

$$\mu_S = \mu_D + \mu_L \quad (14)$$

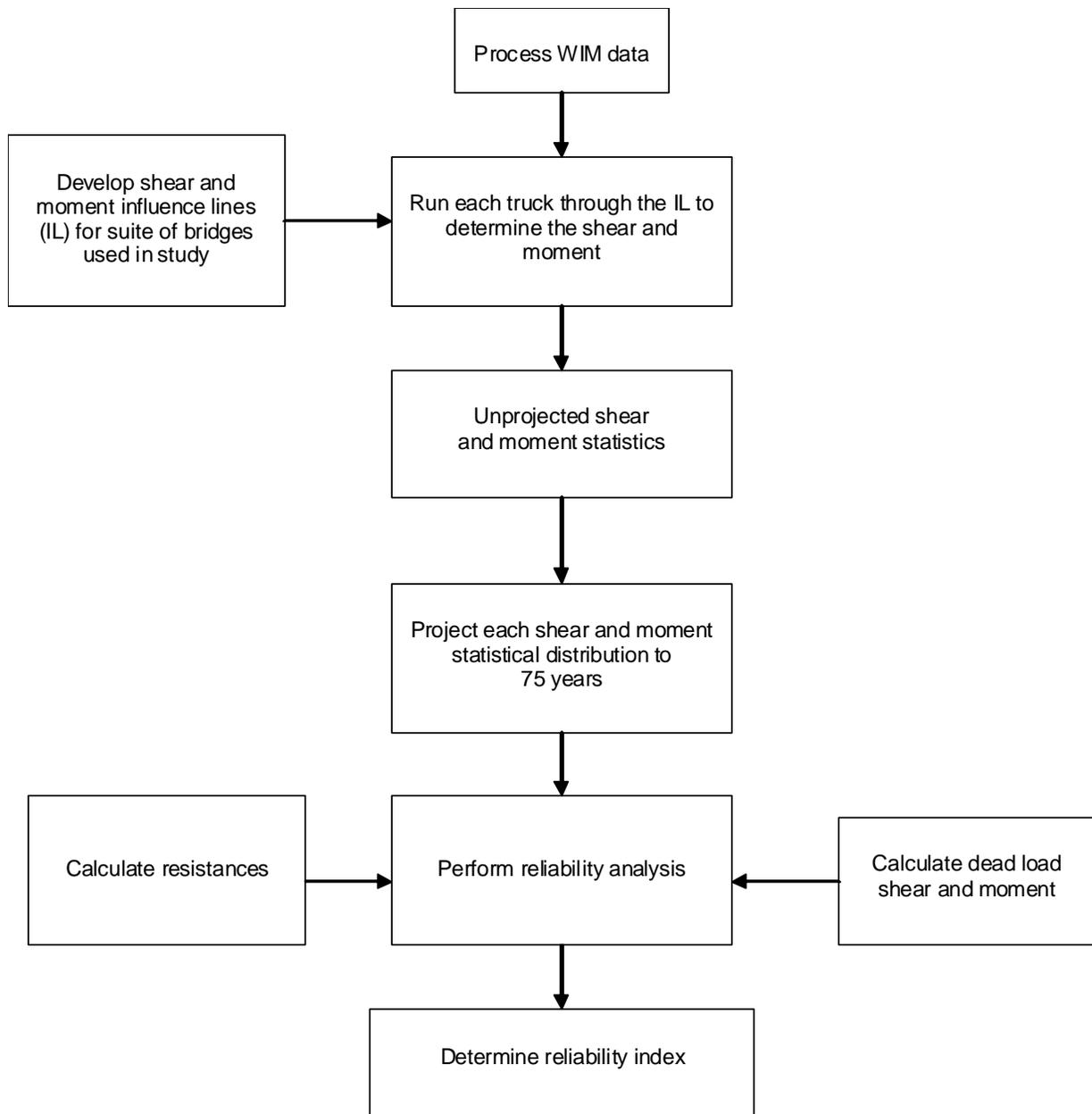
where  $\mu$  indicates the mean. The COV of the total load effect was then be expressed as

$$V_S = \frac{\sigma_S}{\mu_S} \quad (15)$$

The reliability index  $\beta$  defined in Eq. (5), was calculated using the First Order Reliability Method<sup>10</sup>. However, since both the load  $S$  and resistance  $R$  were assumed to be lognormally distributed, the calculation of the reliability index was simplified to

$$\beta = \frac{\ln(\mu_R) - \ln(\mu_S)}{\sqrt{V_R^2 + V_S^2}} \quad (16)$$

where  $\mu_R$  and  $\mu_S$  represent the means of the resistance and total load effect, and  $V_R$  and  $V_S$  are their coefficients of variation, respectively. Figure 2 presents a flowchart of the overall procedure of reliability index analysis.



**Figure 2 Flowchart of reliability index calculation**

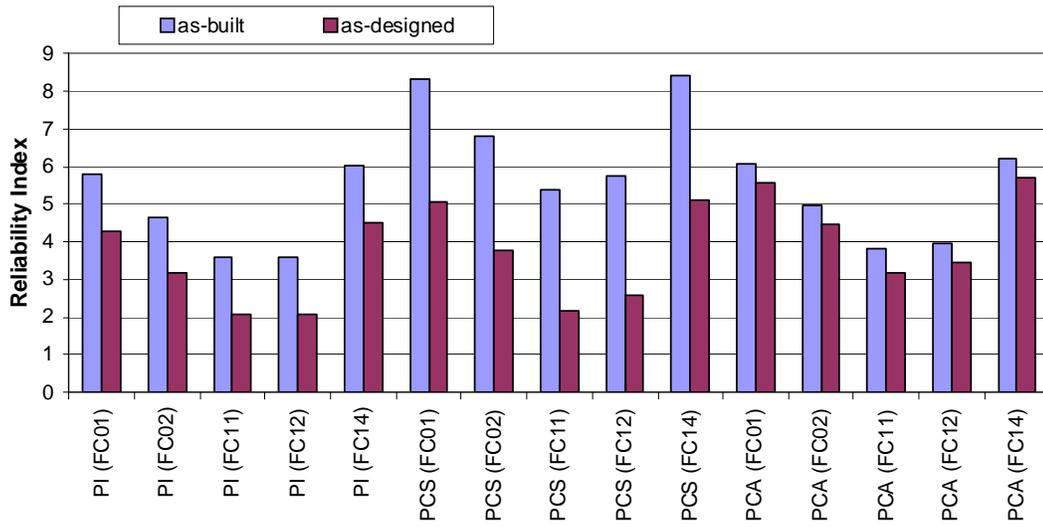
**RELIABILITY ANALYSIS RESULTS**

Reliability indices of the fifteen selected prestressed concrete bridges were calculated following the overall procedure shown in Figure 2. Figures 3 and 4 show the calculated reliability indices with the reliability index  $\beta$  plotted on the vertical axis and the bridge superstructure type for different functional classes plotted on the horizontal axis. For example, in Figure 3, the reliability index  $\beta$  values for moment in PCA for FC14 are 6.2 and 5.7 for the as-built and as-designed conditions, respectively. As discussed earlier, the reliability index calculation model used here refers to only one failure mode: beam flexure or beam shear.

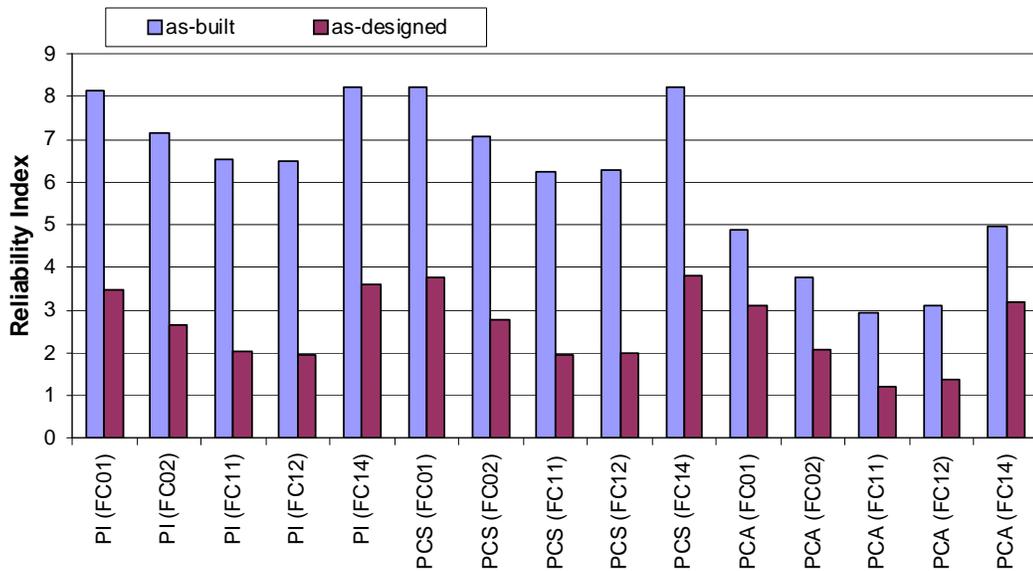
It can be seen from Figures 3 and 4 that the  $\beta$  values ranged from as low as 1.2 to as high as 8.4. The  $\beta$  values for moment ranged between 2.0 and 5.7 for the as-designed case. For the as-built case, the  $\beta$  values for moment ranged between 3.6 and 8.4. In comparison, the  $\beta$  values for shear ranged between 1.2 and 3.8 for the as-designed case and between 2.9 and 8.3 for the as-built case. The results show that the bridges under FC11 and FC12 consistently have the lowest  $\beta$  values. The lower levels of  $\beta$  values may be due to the fact that these two functional classes are in the urbanized areas. This is attributable to the heavier truck loads in this area.

Figures 3 and 4 show significant differences in the reliability index  $\beta$  values between the as-built and the as-designed cases. These differences show the conservatism exercised by individual designers. Small differences are observed in prestressed concrete adjacent box beam bridges while large differences are seen in prestressed concrete I-beam bridges and prestressed concrete spread box beam bridges.

It has been discussed earlier that 3.5 was selected as the target reliability index for component strength in the calibration of the AASHTO LRFD Bridge Design Specifications. This target value also has been suggested to be the threshold of evaluation in this study. Using this threshold, results have shown that there are several cases where  $\beta$  values are lower than 3.5. This indicates that the minimum requirement in the current design code is not adequate, at least for bridges investigated here. Note also that the target value of 3.5 is used for a single structural component (i.e., bridge girder) and not the entire bridge structural system. Hence, a value below 3.5 does not necessarily mean the bridge is unsafe because it is the system, not the component, that determines safety of a bridge.



**Figure 3 Comparison between as-built and as-designed reliability indices for moment**



**Figure 4 Comparison between as-built and as-designed reliability indices for shear**  
**CONCLUSIONS**

It is well known that structural engineers exercise conservatism in their design practice. This often results in additional reserved strength built in the structure. This reserved strength can be sometimes very significant. Large differences in the reliability index  $\beta$  values between the as-built and as-designed in Figures 3 and 4 highlight this fact. Apparently, an extraordinarily large amount of additional reserved strength has been provided beyond what the code calls for. While this conservatism is commonly observed, there is no measure in place to control and/or assure uniform conservatism. Instead, there is always a chance that this conservatism exercised is not adequate to cover the involved risk. For example, in this study,  $\beta$  values lower than the target 3.5 for the as-built case can still be found. This situation deserves adequate attention.

## ACKNOWLEDGEMENTS

The Michigan Department of Transportation (MDOT), the US Federal Highway Administration (FHWA), Roger D. Till, Dr. Gongkang Fu, Dr. John W. van de Lindt, and Yingmin Zhou are gratefully acknowledged for their significant contributions to this study.

## REFERENCES

1. Nowak, A.S. (1999). Calibration of LRFD bridge design code. NCHRP Report 368, Transportation Research Board, National Academy Press, Washington, D.C., U.S.A.
2. Moses, F. and Verma, D. (1987). Load capacity evaluation of existing bridges. NCHRP Report 301, Transportation Research Board, National Academy Press, Washington, D.C., U.S.A.
3. Fu, G. and Hag-Elsafi, O. Vehicular overloads: load model, bridge safety, and permit checking. ASCE Journal of Bridge Engineering, Vol.5, No.1, Feb. 2000, 49-57.
4. AASHTO (1998). "LRFD bridge design specifications. 2nd Ed., Washington, D.C., U.S.A.
5. Ang, A-H.S., and Tang, W.H. (1975). "Probability Concepts in Engineering Planning and Design, I". John Wiley & Sons, New York.
6. Frangopol, D.M. (1999). Bridge Safety and Reliability. American Society of Civil Engineers, U.S.A.
7. Thoft-Christensen, P. and Baker, M.J. (1982) "Structural Reliability Theory and its Applications". Springer-Verlag, Berlin.
8. Traffic Monitoring Guide, U.S. Department of Transportation, Federal Highway Administration, October 1992.
9. AASHTO (1996). "Standard specification for highway bridge design. 16<sup>th</sup> Ed., Washington, D.C., U.S.A.
10. Madsen, H.O., Krenk, S., and Lind, N.C. (1986). "Methods of Structural Safety." Prentice Hall, Inc. Englewood Cliffs, NJ, U.S.A.